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LETTER TO THE EDITOR

RESPONSE OF SMA SUPERELASTIC SYSTEMS UNDER RANDOM EXCITATION

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1. INTRODUCTION

Since it appeared in the 1960s shape memory alloy (SMA) applications are very welcome in industries because of its special characteristics. There are two primary features of SMA which are of interest. The first feature is called the shape memory effect (SME). A shape memory alloy is able to regain its original configuration after it has been deformed by heating the alloy above its characteristic transition temperature. The second is called superelastic effect (SEE). This effect is observed when a strain is imposed on an SMA material at a temperature above A_f (austenite finish). The alloy system can relieve the stress imposed on it by transforming to the thermally unstable martensite and allowing that martensite to strain as it is formed. These two features are shown in Figure 1 [1].

The martensitic transformation is the basic characteristic of shape memory alloys involved in all the unique characteristics of SMA. The martensitic transformation may be simply illustrated by the change of martensite volume fraction with respect to temperature as shown in Figure 2. The four important transition temperatures are martensite finish (M_f), martensite start (M_s), austenite start (A_s), and austenite finish (A_f) [1].

Until now, most of the SMA applications are force and displacement actuators. Rogers (1988) suggested that shape memory alloy fibers could be embedded into conventional composites such as graphite/epoxy to control the structural acoustic radiation/transmission [1]. Since then, many articles concerning SMA use in active vibration control were presented, these including Nie and Yan suggested that SMA could be used to design an intelligent bearing system to control the critical speed of rotating shafts [2]. All the above applications utilize SMA's first unique character: shape memory effect.

In recent years, some articles began to concern the use of SMA superelastic effect in passive vibration control [3, 4]. From Figure 1, we can see that SMA's hysteresis loop ideally provides an energy-absorbing effect and has zero residual strain upon unloading. Contrasting with other rubber or elastomeric materials, SMA has not only a high intrinsic damping, but also a better rigidity. At the same time, the superelasticity of SMA can also be used to improve the impact damage tolerance of composite materials. All these make SMA an effective metal alternative to rubber-based machinery isolation mounts. If conditions permit, the superelastic behavior could be used combined with the SME effect and provides system with active control features.

The objectives of SMA superelasticity studies mostly concentrated on eliminating the vibration caused by seismic random loading. In this article, response of the system with superelastic behavior under stationary random white-noise excitation is studied by two

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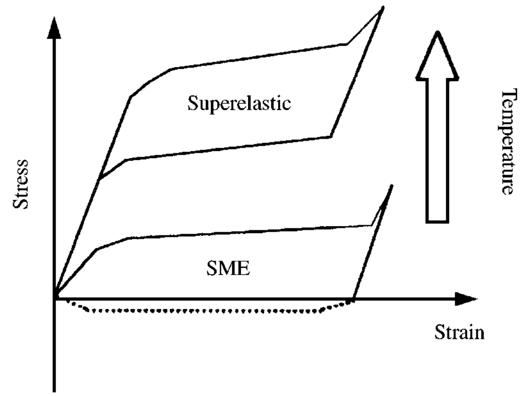

Figure 1. Schematic diagram of SMA stress–strain relation.

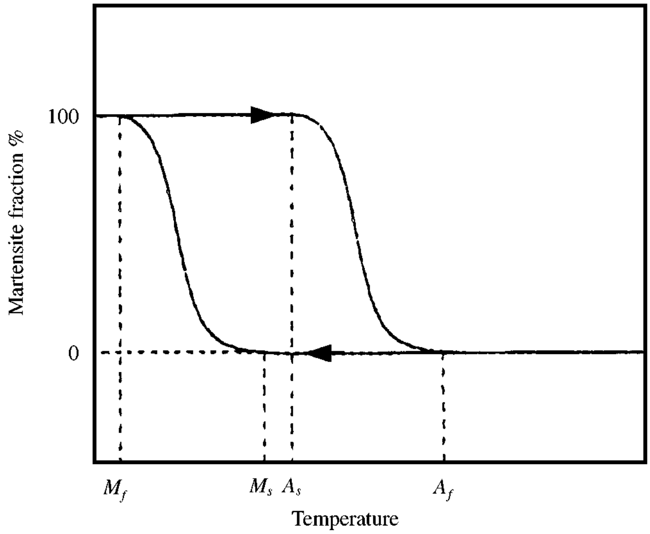

Figure 2. Diagram of martensite fraction versus temperature.

ways: the Monte Carlo method and the equivalent linearization method. The Monte Carlo method is based on Graesser's hysteretic model and the latter one is based on a newly proposed simple superelastic model which has a potential use from the point of view of engineering.

2. NUMERICAL SIMULATION OF SMA SUPERELASTICITY

Characterization of SMA behavior is accomplished here by using Graesser hysteretic model. The model is based on the Bonc–Wen model [5] and introduces a term to describe SMA superelasticity. Figure 3 shows the one-dimensional stress-strain relation described by the model (dashed line) versus experiment result (the material used is SMA Nitinol) [4]. The one-dimensional form of the model is as follows (in order to be consistent with later

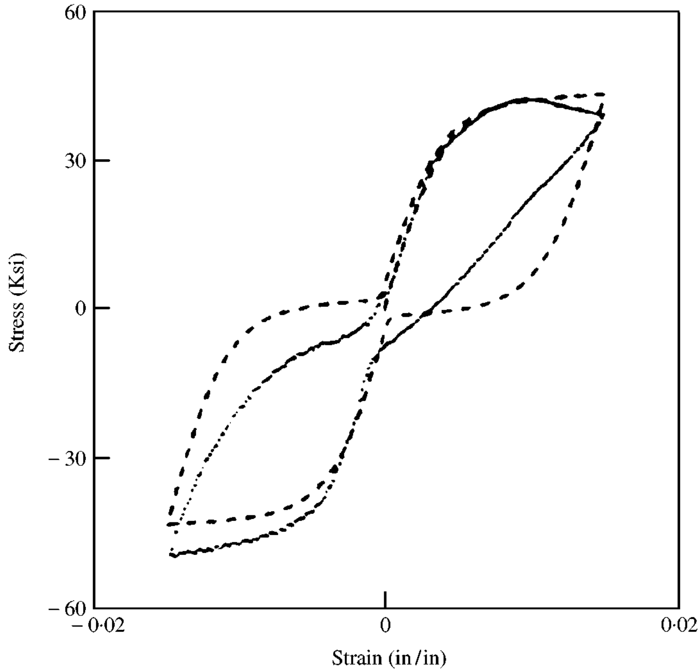

Figure 3. Graesser model (dashed line) versus experiment.

development, we use z to denote hysteretic restoring force and x displacement):

$$\dot{z} = k\left[\dot{x} - |\dot{x}|\left|\frac{z-\beta}{Y}\right|^{n-1}\left(\frac{z-\beta}{Y}\right)\right], \tag{1}$$

$$\beta = k\alpha\left[x - \frac{z}{k} + f_T|x|^{c'}\operatorname{erf}(a'x)\right], \tag{2}$$

where z is the one-dimensional hysteretic force, x the one-dimensional displacement, k the system stiffness when SMA is an austenite state, Y the threshold force to start stress-induced phase transformation (analogous to yield force) at a specific reference temperature, α a constant which determines the slope of the inelastic region $= k'/k$, where k' is the system stiffness when SMA is the martensite state, a' a constant controlling the amount of elastic recovery during unloading, f_T a constant controlling the type and size of hysteresis (superelasticity or twinning hysteresis, for detailed information, see reference [6]), n a constant controlling the sharpness of transition from elastic to inelastic behavior, c' a constant controlling the slope of the unloading force plateau, $(\dot{\cdot})$ the ordinary time derivative, $|x|$ the absolute value of x and $\operatorname{erf}(x)$ the error function of the argument x,

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt. \tag{3}$$

It is worth pointing out that when the third term on the right-hand side of equation (2) is dropped (i.e., when $f_T = 0$), the above model has the same essence as the Bonc–Wen model which can be used to describe another SMA hysteresis called twinning hysteresis at a temperature below M_f and also existed in most of the structures.

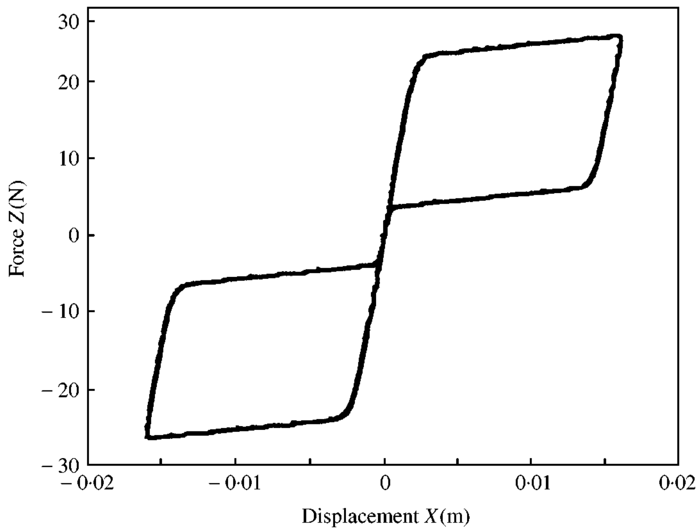

Figure 4. Cyclic response of SMA superelastic model $x = 0{\cdot}016\ \sin t$: $k = 1{\cdot}0 \times 10^4$ N/m, $Y = 10$ N, $n = 3$, $\alpha = 0{\cdot}0197$, $a' = 2500$, $c' = 0{\cdot}001$.

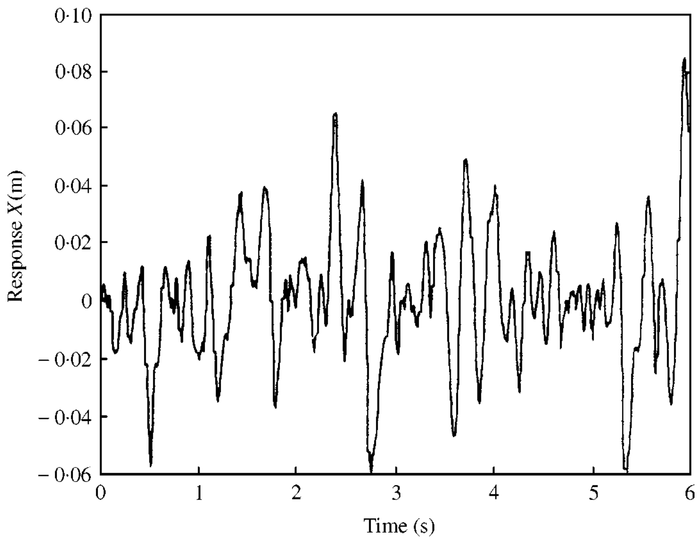

Figure 5. Response of SMA superelastic system under random white-noise excitation.

Based on the above model, the computation program was written by use of the high order R–K method to calculate the response of SMA superelastic system under external loading. For the purpose of model demonstration, a sine wave force is applied to the superelastic system and the force–displacement relation is shown in Figure 4, which shows similarity to Graesser's calculation. Under random excitation, the program also converges well. The response of the system under stationary white noise is shown in Figure 5 and force–displacement relation is shown in Figure 6. Parameter values of the one-d.o.f. system used in this article are: system mass, $m = 1{\cdot}0$; viscous damping ratio; $\xi_0 = 0{\cdot}02/0$; system stiffness, $k = 1{\cdot}0 \times 10^4$; the power-spectral density of the stationary white noise,

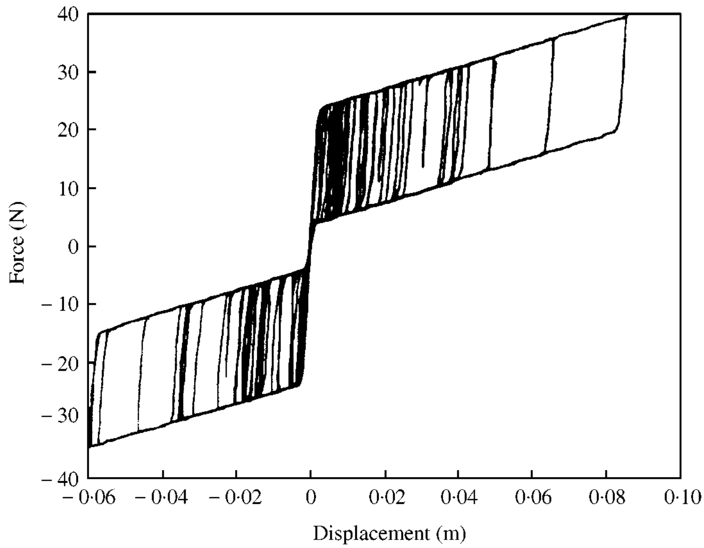

Figure 6. Force–displacement relation due to random white-noise excitation.

$S_0 = 10/(2\pi)$. Other parameters used in the Graesser model are seen in Figure 4. All the parameter values are expressed in SI units.

3. METHOD OF EQUIVALENT LINEARIZATION

Besides the above Monte-Carlo method, there are still several methods to predict response statistics in the non-linear system under random excitation, such as Markov method, equivalent linearization method, perturbation methods, functional series representations, etc. [7, 8]. Among these, the equivalent linearization method is the most popular method in engineering especially for hysteretic systems. In this study, we use this method to study the superelastic system's response under random excitation.

First, we try to use the previous Graesser model in the equivalent linearization method (for details of the method, refer to Wen's work [5, 9].) In the derivation course, the equation of motion of the system cannot be linearized directly in closed form, i.e., the coefficients of the linearized system cannot be obtained exactly as simple algebraic functions of the response variable statistics. In order to get the coefficients, numerical methods were used to evaluate the triple integral. The RMS response σ_x gave very poor accuracy compared with the previous Monte-Carlo simulation because of the error caused by numerical triple integral computation.

The purpose of the following study is to present a new simple superelastic model which can get a close form for the coefficients of the linearized system using equivalent linearization. The accuracy of the result calculated using the model is verified against the above Monte-Carlo simulations for all ranges of response levels.

Following references [5] and [9], the restoring force $g(x, \dot{x})$ in the superelastic system is described by

$$g(x, \dot{x}) = \alpha k x + (1 - \alpha) k z, \tag{4}$$

in which the first part on the right-hand side is a non-hysteretic component, and α, x, k are as stated before, z in the second part, is a hysteretic component, a function of the time

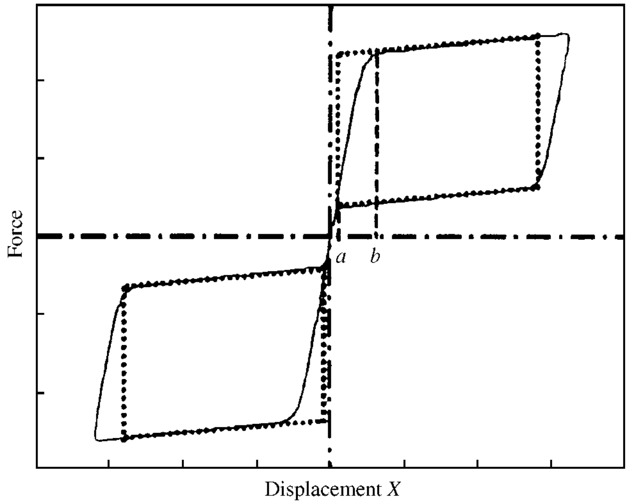

Figure 7. A simple linear model used in this study compared with Graesser model. ——, Graesser model; •••••• Present model.

history of x. In this study, z is expressed as the following simple linear equation (as shown in Figure 9):

$$z = \{1 - \text{sign}[\text{sign}(|x| - a) + 1]\}x + \frac{\text{sign}(|x| - a) + 1}{2}\left[\frac{\text{sign}(x) + \text{sign}(\dot{x})}{2}(b - a) + a\,\text{sign}(x)\right], \qquad (5)$$

in which sign(x) gives $-1, 0$ or 1 depending on whether x is negative, zero, or positive, a and b represent the elastic limit and the point with which to begin martensite phase transition, respectively (as shown in Figures 7 and 8), which can be determined by experiment.

According to the equivalent linearization method, under the condition that the mean square error in replacing equation (5) by the equation of a linear system:

$$z = C_e\dot{x} + K_e x \qquad (6)$$

can be minimized if the elements C_e and K_e are given by

$$\frac{\partial E(e)^2}{\partial C_e} = 0 \qquad \frac{\partial E(e^2)}{\partial K_e} = 0, \qquad (7)$$

in which $E[\,]$ denotes the expected value, and e denotes the error of substituting equation (6) for equation (5) (Figure 9).

Solving equation (7), we have

$$C_e = \frac{(b - a)}{\sqrt{2\pi}\sigma_{\dot{x}}}\left[1 - \text{Erf}\left(\frac{a}{\sqrt{2}\sigma_x}\right)\right],$$

$$K_e = \frac{(a + b)}{\sqrt{2\pi}\sigma_x}\,e^{-a^2/2\sigma^2}. \qquad (8)$$

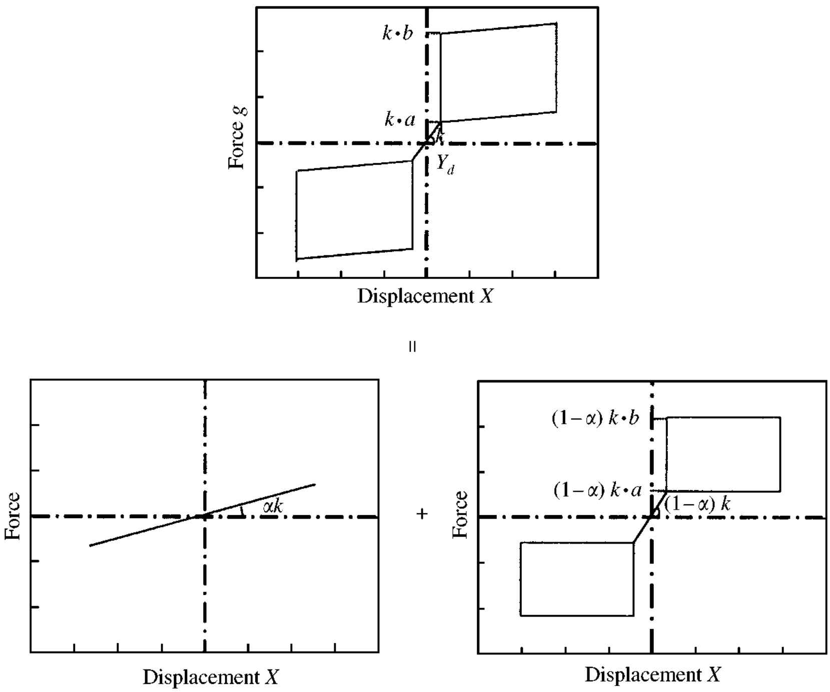

Figure 8. Schematic diagram of decomposition of the restoring force.

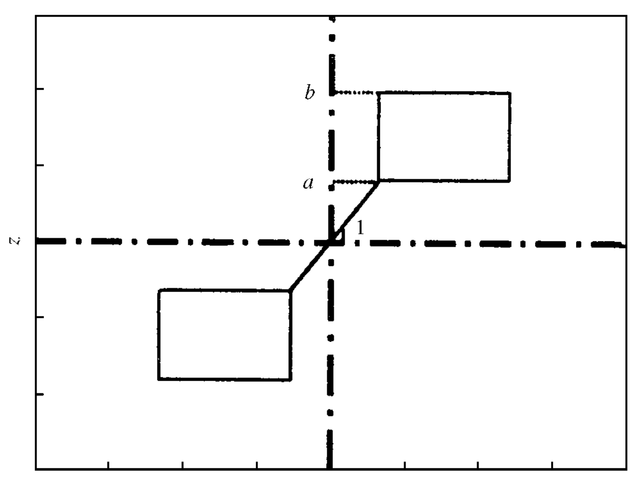

Figure 9. A simple superelastic model.

Therefore, the governing equations of motion of the equivalent linear system are

$$\ddot{x} + 2\xi_0\omega_0\dot{x} + \alpha\omega_0^2 x + (1-\alpha)\omega_0^2 z = F(t)/m,$$

$$z = C_e\dot{x} + K_e x, \tag{9}$$

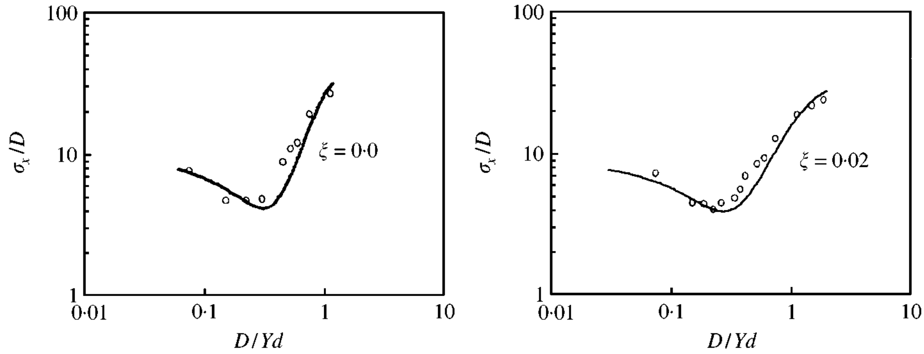

Figure 10. Non-dimensional r.m.s. response of system with different damping ratios and comparison with Monte-Carlo solution. ——, Linearization; ○, Simulation.

in which ξ_0 is the viscous damping ratio, ω_0 the natural frequency of the system, m the system mass, $F(t)$ the stationary white-noise excitation with intensity $I = 2\pi S_0$. The solution of equation (9) is identical to other non-linear problems using the equivalent linearization method.

Systems with different damping ratios are calculated by the equivalent method. The r.m.s. response σ_x as a function of the excitation level is shown in Figure 10. σ_x is normalized by $D = \sqrt{2S_0/\omega_0^3}$ and the excitation level is indicated by the non-dimensional quality D/Y_d, where Y_d denotes the displacement corresponding to force Y (shown in Figure 8).

The r.m.s. response for various excitation levels with analytical solution is compared with the simulated result. The agreement is good for all response levels. The small scatter can be attributed to the difference between the two models. The Graesser model which is more realistic is simplified in this study, but gets more accurate results. From the point of view of engineering, the accuracy of the equivalent linearization method is high enough.

4. SUMMARY AND CONCLUSION

Two methods were used to predict superelastic system's response under stationary white-noise excitation. The Monte-Carlo method is based on the Graesser model and the equivalent linearization method is based on a newly proposed simple model. The advantage of the simple model is that the motion equation of the systems is linearized directly in closed form. No literature has concern about superelastic system's response under random excitation. The results got from the two methods are compared and show good agreement in all excitation levels.

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